Use the graph at right to explore similarity in the coordinate plane.

1. Write down the vertices of ΔPQR.

   $P($ 0 , 4 $)$, $Q($ 0 , 0 $)$, $R($ 2 , 0 $)$

2. Multiply each coordinate of each vertex of ΔPQR by 3. Then graph $\Delta P'Q'R'$ with these new vertices. How is $\Delta P'Q'R'$ related to ΔPQR ?

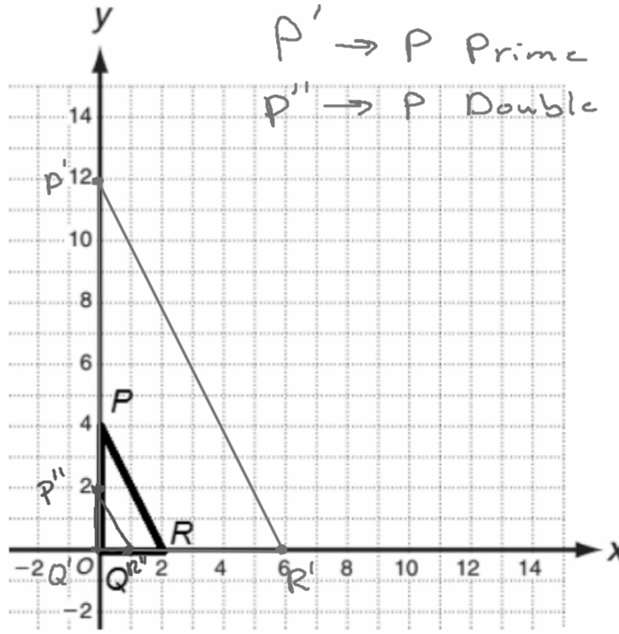
   $P'($ 0 , 12 $)$, $Q'($ 0 , 0 $)$, $R'($ 6 , 0 $)$

3. Now multiply each coordinate of each vertex of ΔPQR by ½ . Then graph $\Delta P"Q"R"$ with these new vertices. How is $\Delta P"Q"R"$ related to ΔPQR ?

   $P"($ 0 , 2 $)$, $Q"($ 0 , 0 $)$, $R"($ 1 , 0 $)$

P′ → P Prime

P″ → P Double Prime

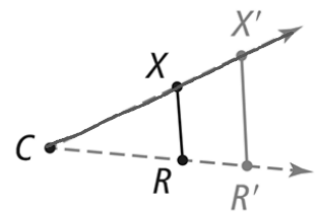


Dilations

origin

A dilation $D_{(n,\ C)}$ is a transformation that has center of dilation C and scale factor n, where $n > 0$, with the following properties:

- Point R maps to R' in such a way that R' is on $\overrightarrow{CR}$ and $CR' = n \cdot CR$.
- Each length in the image is n times the corresponding length in the preimage (i.e., $X'R' = n \cdot XR$).



- The image of the center of dilation is the center itself (i.e., $C' = C$).
- If $n > 1$, the dilation is an *enlargement.*
- If $0 < n < 1$, the dilation is a *reduction.*
- Every angle is congruent to its image under the dilation. ✓

On a coordinate plane, the notation D_n describes the dilation with the origin as center of dilation.

Preimage ⟶ Image

Start with

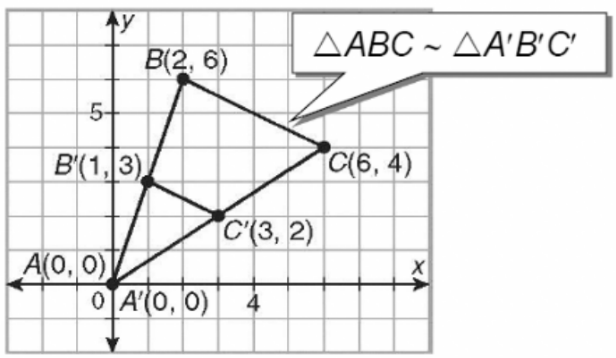
After Dilation

Scale Factor → Ratio of Sides (K)

Scale Factor > 1 Enlargement
(Gets Bigger)

Scale Factor $0 < K < 1$ Reduction
(Gets Smaller)

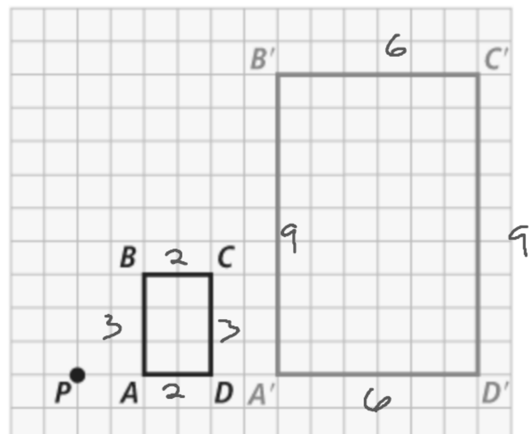
A dilation is a transformation that changes the __Size__ of a figure but not its __Shape__. The preimage and image are always similar. A __Scale Factor__ describes how much a figure is enlarged or reduced.



Example 1. Triangle ABC above has vertices $A(0, 0)$, $B(2, 6)$, and $C(6, 4)$. Find the coordinates of the vertices of the image after a dilation with a scale factor ½ .

Preimage ΔABC		**Image** $\Delta A'B'C'$
$A(0,0)$	$\rightarrow (\frac{1}{2}\cdot 0,\quad \frac{1}{2}\cdot 0) \rightarrow$	$A'(0,0)$
$B(2,6)$	$\rightarrow (\frac{1}{2}\cdot 2,\quad \frac{1}{2}\cdot 6) \rightarrow$	$B'(1,3)$
$C(6,4)$	$\rightarrow (\frac{1}{2}\cdot 6,\quad \frac{1}{2}\cdot 4) \rightarrow$	$C'(3,2)$

Rectangle $A'B'C'D'$ is a dilation with center P of $ABCD$. How are the side lengths and angle measures of $ABCD$ related to those of $A'B'C'D'$?



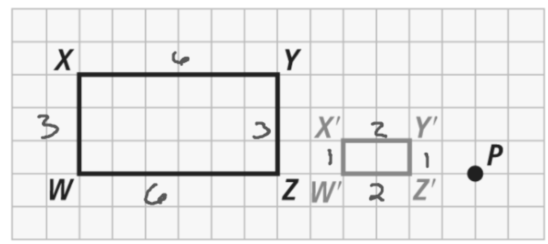
SOLUTION

$k = \frac{3}{1}$

$\frac{6}{2} = \frac{9}{3} = \frac{6}{2} = \frac{9}{3}$

$\frac{3}{1}$

2. Rectangle $W'X'Y'Z'$ is a dilation with center *P* of *WXYZ*. How are the side lengths and angle measures of the two figures related?



Enter your answer

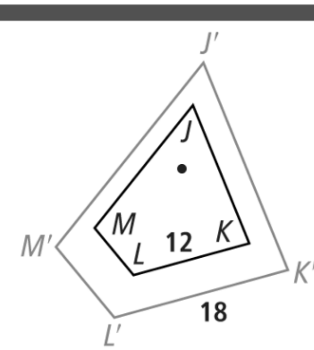
$\frac{1}{3} = \frac{2}{6} = \frac{1}{3} = \frac{2}{6} =$

$k = \frac{1}{3}$

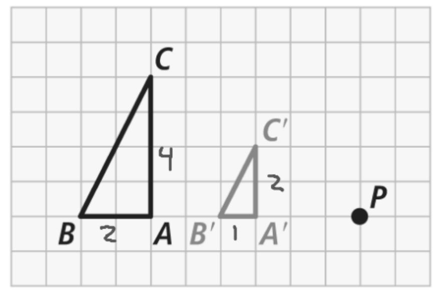
Quadrilateral $J'K'L'M'$ is a dilation of $JKLM$. What is the scale factor?

$k = \frac{18}{12} = \frac{3}{2}$

SOLUTION



3. Consider the dilation shown.



a. Is the dilation an enlargement or a reduction?

Enter your answer.

b. What is the scale factor?

$K = \frac{1}{2}$

Dilate a Figure With Center at the Origin

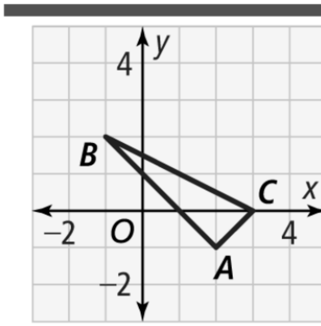
What are the vertices of $D_3(\triangle ABC)$?

The notation $D_3(\triangle ABC)$ means the image of $\triangle ABC$ after a dilation centered at the origin, with scale factor 3.

SOLUTION

A (2,-1)
B (-1,2)
C (3,0)

A' (6,-3)
B' (-3,6)
C' (9,0)

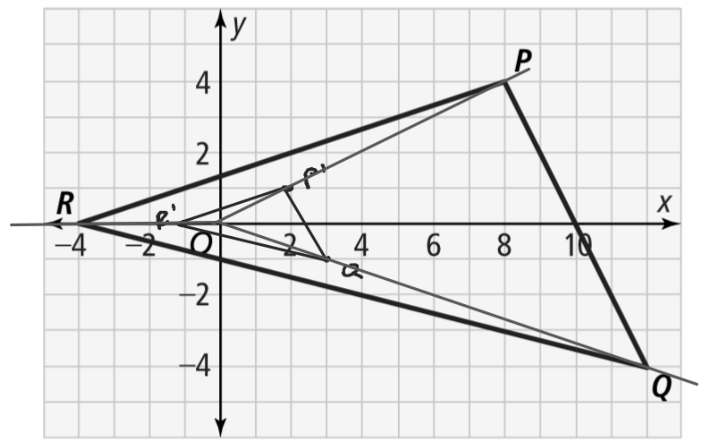


4. Use $\triangle PQR$.

$P(8,4)$
$Q(12,-4)$
$R(-4,0)$

a. What are the vertices of $D_{\frac{1}{4}}(\triangle PQR)$?

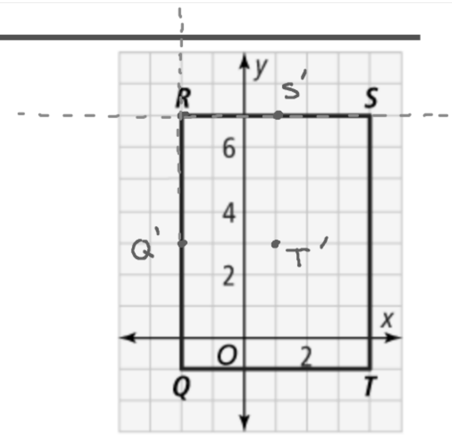
$P'(2,1)$
$Q'(3,-1)$
$R'(-1,0)$



b. How are the distances to the origin from each image point related to the distance to the origin from each corresponding preimage point?

$OP' = \frac{1}{4}OP$

What are the vertices of $D_{(\frac{1}{2},\ R)}(QRST)$?



SOLUTION

R (0,0)

S (6,0) → S'(1,7)

Q (0,-8) → Q'(-2,3)

T (6,-8) → T'(1,3)

Preimage Point	Change From R(-2,7)		Half of the Change from R(-2,7)		Add to R (-2, 7)	Image Point
	Horiz	Vert	Horiz	Vert		
Q (-2, -1)	0	-8	0	-4	(-2,3)	
S (4, 7)	6	0	3	0	(1,7)	
T (4, -1)	6	-8	3	-4	(1,3)	

AB = A(-2,-1) B(-3,1)

$\sqrt{(-3+(2))^2+(1+1)^2}$

$\sqrt{(-1)^2+(2)^2}$

$\sqrt{5}$

A'(-2,-1)

B'(-3.5,2)

5. A dilation of $\triangle ABC$ is shown.

a. What is the center of dilation?

Enter your answer.

A'

AB =

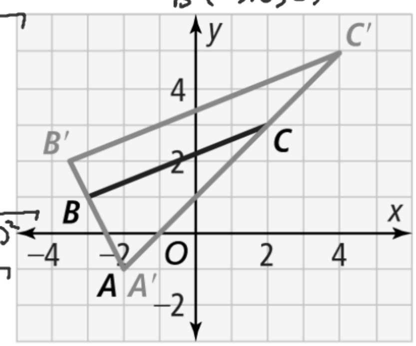
A'B' =

$A'B' = \sqrt{(-3.5+2)^2+(2+1)^2}$

$\sqrt{(-1.5)^2+3^2}$

$\sqrt{2.25+9}$

$\sqrt{11.25}$

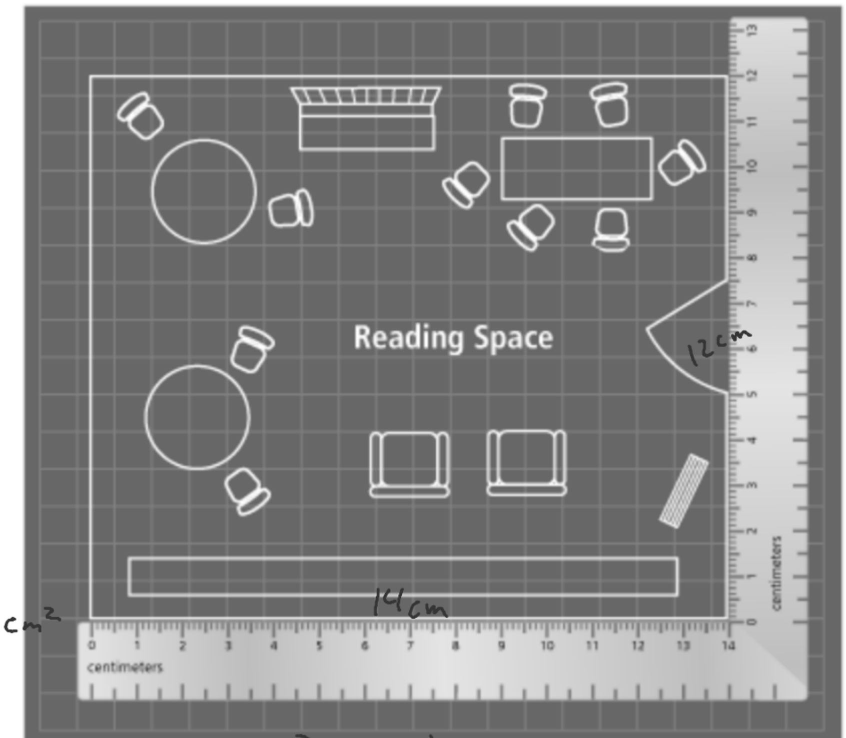


$K = \frac{AB}{A'B'}$ $\frac{A'B'}{AB}$

$\frac{\sqrt{5}}{\sqrt{11.25}}$ $\frac{\sqrt{11.25}}{\sqrt{5}}$

b. What is the scale factor?

A blueprint for a new library uses a scale factor of $\frac{1}{50}$. Mr. Ayer measures the reading space on the blueprint to find the actual dimensions and area so he can order furniture.



A. What are the actual dimensions of the reading space?

$\frac{1}{50} = \frac{14}{x}$ $\qquad$ $\frac{1}{50} = \frac{12}{y}$

$Y = 700$ $\qquad$ $y = 600$

B. What is the actual area of the reading space? How does the actual area relate to the area on the blueprint?

$A = (700)(600) = 420{,}000 \text{ cm}^2$

$A = 168 \text{ cm}^2$

$\left(\frac{1}{50}\right)\left(\frac{1}{50}\right) = \frac{1}{2500}$

6. A blueprint for a house uses a scale factor of $\frac{1}{20}$.

a. If the dimensions of the actual kitchen are 3.1 m by 3.4 m, what are the dimensions of the kitchen on the blueprint?

$\frac{1}{20} = \frac{3.1}{x}$ $\frac{1}{20} = \frac{3.4}{y}$

62 m 68 m

b. What is the relationship between the area of the actual kitchen and the area of the kitchen on the blueprint?

$A = (62)(68) = 4216$

Blue Print

$A = (3.1)(3.4) = 10.54$